Material Point Method Modelling of Landslides with Coupled Segregation

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Abstract Landslides, debris flows and avalanches often exhibit strong segregation during flow and deposition. The largest particles are usually found at the nose of the avalanche, with moderate sized particles at the free surface, and smaller particles at the base of the flow. At the same time, we know that the constitutive behaviour of such a system is strongly influenced by the local average grainsize. In numerical modelling of these flows, the coupling of the spatial heterogeneity and constitutive behaviour has heretofore only been weakly coupled, if addressed at all. Here, we will present a unified framework for coupling the feedback between these two phenomena using the material point method. Several examples of landslide propagation will be investigated. The effect of flow lubrication via segregation will be highlighted.

1 Introduction

Granular flows in nature, such as landslides, debris flows and avalanches, often are composed of particles ranging in size from clay platelets to boulders. These particles are constructed from a variety of materials, with significantly varying properties. Unifying the description of these flows has proven to be a considerable challenge, not least because of the spatial variability of the material [1].

One significant issue controlling the spatial variability of these flows is grainsize segregation [2]. In the context of a gravity current, this phenomenon causes larger particles to rise to the surface of the flow, and smaller particles to sink to the base. A typical phenomenological description of this mechanism is that after a collision between particles, a new void space is formed, which is preferentially filled by a smaller particle falling into, as it is less likely that a large particle will fit into the void than a small one.

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Whilst the physical mechanisms responsible for this phenomenon are still under investigation [3], many analytic [4–6] and numerical [7–9] models exist. These models, however, generally treat separately the bulk flow and the segregation induced flow, with some recent notable exceptions [10, 11]. This lack of coupling between the bulk flow and the segregation prevents a systematic study of the bulk rheology of the material as it segregates. Here, we develop a comprehensive framework for studying grainsize evolution that is weakly coupled to the bulk rheology. This is then implemented in a large deformation continuum solver, using the material point method (MPM), which is distributed under as open source code (under the GPL3.0 license) and freely available at http://www.benjymarks.com.

2 Grainsize Dynamics

Representing a polydisperse granular material as a five dimensional continuum has been the subject of several previous studies [8, 12], and for brevity only the salient details will be discussed here. The novelty of this method is the inclusion of an additional coordinate, s, into the continuum fields, which is used to represent the grainsize distribution. For example, the concentration of each grainsize $\phi(s)$, can easily be stated. Following [13], and assuming that particles do not expend or shrink, mass and momentum conservation in the entire domain can be written in a local form as:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = \mathbf{F}. \tag{2}$$

where ρ is the partial density of the material, $\mathbf{F}$ the total force per unit volume and $\mathbf{u}$ the velocity field. In a recently submitted manuscript [12], a significant development was the formulation of a relationship between the segregation velocity, $\hat{\mathbf{u}}$ and the bulk kinetic stress field, $\bar{\boldsymbol{\sigma}}_k$, as

$$\hat{\mathbf{u}} = \frac{1 - \bar{s}/s}{\bar{\rho} c_0} \nabla \cdot \bar{\boldsymbol{\sigma}}_k. \tag{3}$$

where s is the grainsize, $\bar{s}$ is the local mean grainsize, $\bar{\rho}$ is the bulk density and c_0 is a rate which controls the rate of segregation.

This relationship allows us to describe the segregation phenomenon, once the kinetic stress field is known. This field, which is closely related to the granular temperature T_g, has recently been described using its own set of evolution equations [14]. Here, we merely assume that the granular temperature scales to first order as the rate-of-shear tensor, $\mathbf{S}$, following [15], such that $\bar{\boldsymbol{\sigma}}_k \approx c_1 \bar{\rho} \mathbf{S}$, where c_1 is a free parameter (Fig. 1).

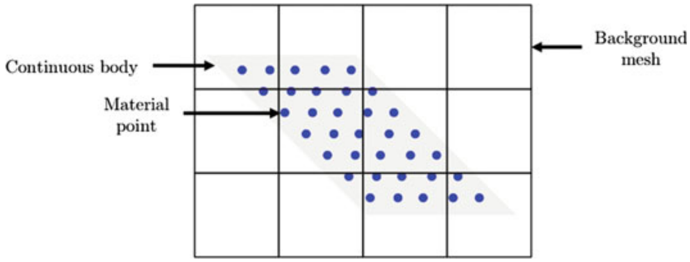

Fig. 1 Schematic diagram of the material point method. A continuous body, represented by the *gray* region, is discretised onto a set of material points (in *blue*). At each time step, information is projected from the material points to the background mesh (shown in *black*). Continuum equations are then solved on the background mesh, and the updated quantities projected back to the material points

3 Material Point Method Implementation

A material point method code is implemented following [16], with additional parameters included to describe the segregation process. MPM has successfully been used to model a variety of large deformation problems, from the macro scale (whole landslides) [17] to the micro scale (modelling granular materials) [18]. There are at least two options available for impregnating such a continuum solver with a grainsize distribution. One option would be to add material points for each phase of material, such that if a bidisperse material was modelled, there would be two distinct material points, each carrying a proportion of the total mass. This method suffers from a significant increase in computational time (for arbitrarily large polydispersity, the computational time also scales arbitrarily large). This method also has difficulty dealing with particle breakage (which is not explicitly modelled here), as material points must lose mass, and new particles must be added to represent fragments. A second option involves describing the grainsize distribution within each material point as a discrete histogram of grainsize (or alternatively an analytic function, if required). This is the method pursued here.

The grainsize distribution is discretised into N_s components, such that the solid fraction in a particular grainsize bin $s_a \leq s < s_b$ can be expressed as $\Phi(s_i) = \int_{s_a}^{s_b} \phi(s)\, ds$. This solid fraction is stored as an additional vector for each material point, and at each time step is mapped from the material points to the background mesh, along with the mass and momentum as:

$$m_i^n = \sum_p w_{ip}^n m_p, \tag{4}$$

$$m_i^n \mathbf{u}_i^n = \sum_p w_{ip}^n m_p \mathbf{u}_i^n, \tag{5}$$

$$m_i^n \boldsymbol{\Phi}_i^n = \sum_p w_{ip}^n m_p \boldsymbol{\Phi}_p^n, \tag{6}$$

where n is the current time step, i the grid points, p the material points and w_{ip} the interpolation weights. During the update stage of the MPM, a further evolution equation is solved for $\boldsymbol{\Phi}$, as

$$\frac{\partial \boldsymbol{\Phi}}{\partial t} + \nabla \cdot (\boldsymbol{\Phi} \hat{\mathbf{u}}) = 0, \tag{7}$$

which is the typical statement of conservation of grainsize [4], but extended from bidisperse to polydisperse materials. Due to the non-linear nature of this equation [5], and the requirement that for perfect segregation there must exist discontinuities in the solution, we require a sophisticated numerical solution scheme that is capable of properly accounting for this behaviour. The advective fluxes are solved for using a high-resolution central scheme that is total variation diminishing, as described in [19]. Putting together (7) and (3), we recover the governing equation for segregation in this system:

$$\frac{\partial \boldsymbol{\Phi}}{\partial t} + \nabla \cdot \left(\boldsymbol{\Phi} c \left(1 - \frac{\bar{s}}{s} \right) \nabla \cdot \mathbf{S} \right) = 0, \tag{8}$$

where $c = c_1/c_0$. Using the formulation SD2 from [19], incremental changes of $\boldsymbol{\Phi}$, termed $\Delta\boldsymbol{\Phi}_i^{n+1}$, can be computed. The final step required to complete the simulations is a mapping back from the nodal grainsize distribution $\boldsymbol{\Phi}_i^n$ to the new values at the material points. This is done in the same manner as for position, $\mathbf{x}$ and velocity, $\mathbf{u}$, by solving

$$\mathbf{u}_p^{n+1} = \mathbf{u}_p^n + \sum_i w_{ip} \mathbf{a}_i^n \Delta t, \tag{9}$$

$$\mathbf{x}_p^{n+1} = \mathbf{x}_p^n + \sum_i w_{ip} \mathbf{u}_i^{n+1} \Delta t, \tag{10}$$

$$\boldsymbol{\Phi}_p^{n+1} = \boldsymbol{\Phi}_p^n + \sum_i w_{ip} \Delta\boldsymbol{\Phi}_i^{n+1}, \tag{11}$$

where $\mathbf{a}_i^n$ is the acceleration calculated from conservation of momentum at each grid point.

4 1D Flow

An initial test was conducted for the case of one dimensional laminar flow down an inclined plane, with a rough basal plane that maintains $\mathbf{u} = 0$ at the base (but not $\hat{\mathbf{u}} = 0$). Initially, the material is at rest, and time $t = 0$, gravity of 9.81 m/s is applied at an angle of 18 degrees. A two dimensional system is modelled, with width of 2 grid cells in the x-direction (parallel to the flow) and 51 grid cells in the y-direction (perpendicular to the slope), with the grid spacing in both directions equal to 0.02 m, such that the flow is 1 m deep. In each grid cell, we initially seed 9 regu-

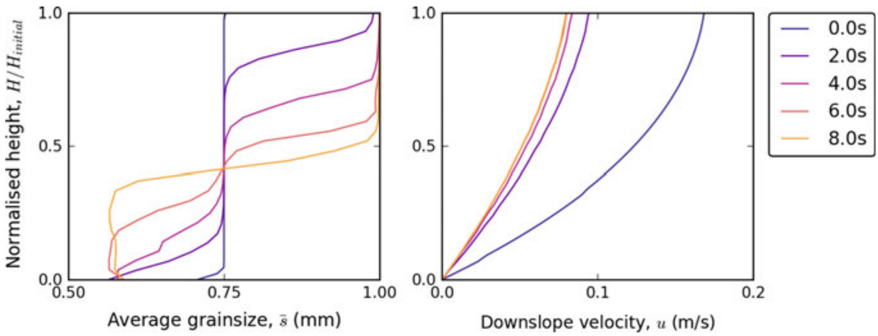

Fig. 2 Changing average grainsize in a one dimensional laminar flow down an inclined plane

larly spaced material points, each with equal mass. The initial grainsize distribution of each material point is chosen to be bidisperse, with 50% each of small and large particles (nominally 0.5 and 1 mm, but note that only their relative sizes affect the segregation velocity). A material density of 1000 kg/m^3 is used. This system models a bidisperse fluid, initially homogeneous in space and in grainsize distribution.

We introduce rheological coupling by allowing the material properties to depend on the grainsize distribution. For illustrative purposes, this work has been conducted using a newtonian rheology ($\tau = \mu\dot{\gamma}$), where the newtonian shear stress is proportional to the average grainsize, as $\mu_s = 10^3\bar{s}$, such that material points composed of primarily small particles will flow faster than larger ones. This is not designed to be a rigorous test of the rheology of landslides.

The results of this test are shown in Fig. 2. It is observed that over time, large particles segregate towards the free surface, and small particles collect at the base of the flow. This is accompanied by a slight change in rheology.

5 Conclusions

An initial investigation has shown that it is possible to couple bulk flow and segregation dynamics using the material point method. By solving the segregation equations directly on the background mesh, it is relatively straightforward to include these effects. Once segregation is included in the description of the flow, rheological coupling is trivial to model, given that there is a known relationship between the grainsize distribution and the constitutive behaviour of the material. In the future, this method will be used to study unsteady, two and three dimensional problems, such as the formation of granular bores, and levee deposits, where in natural systems significant segregation is observed [20, 21].

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