Polydisperse Segregation Down Inclines: Towards Degradation Models of Granular Avalanches

Benjy Marks, Itai Einav, and Pierre Rognon

Abstract Segregation is a well known yet poorly understood phenomenon in granular flows. Whenever disparate particles flow together they separate by size, density and shape. If we wish to know how to separate particles more efficiently, or even how to keep them mixed together, we require a good understanding of both the phenomenology of the flow, and a quantitative analysis of the evolving particle size distribution towards a steady state. This chapter outlines the continuing effort towards this end, and provides a clue as to the future direction of our research.

Keywords Granular materials • Segregation • Avalanches • Kinetic sieving • Grain size distribution

Many forms of segregation occur in different geometries, with varying granular materials and flow regimes (Jaeger et al. 1996; Knight et al. 1993; Makse et al. 1997). A summary of such segregative phenomena is described in Hutter and Rajagopal (1994) and Ottino and Khakhar (2000). We deal here with the segregation due to size and density differences in particles undergoing shallow free surface flow down an inclined plane.

This type of segregation occurs in natural avalanches, causing small particles to migrate to the base of the flow. These small particles create a lubrication layer which speeds up the flowing layers above it, creating higher shear rates and more crushing. To be able to model an avalanche, we first need to understand how a polydisperse flow segregates before we can investigate the role of degradation in the flow.

In general, research on segregation is studied in a bi-mixture framework. This is a useful path to capture the underlying mechanisms related to industrial separation processes, which often tend to deal with only two grainsizes. Nevertheless, it should

B. Marks (✉) · I. Einav · P. Rognon
School of Civil Engineering, The University of Sydney, Sydney, NSW 2006, Australia
e-mail: benjy.marks@sydney.edu.au; itai.einav@sydney.edu.au; pierre.rognon@sydney.edu.au

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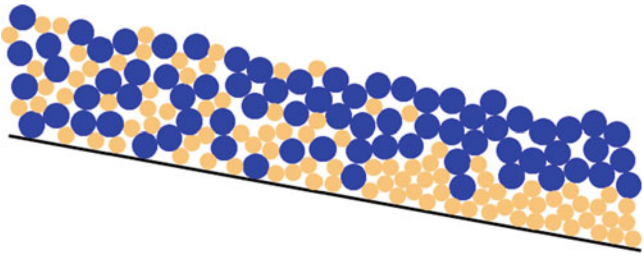

Fig. 1 Effect of kinetic sieving on particles flowing down an inclined slope from *left* to *right*. At the *left*, the particle size distribution is constant with height, while at the *right* it varies with height. This arrangement of sizes, with large particles at the top and smaller particles at the base of the flow is termed inverse grading

be emphasised that even in such problems the idea of identifying only two species is restrictive, since this depends on the tolerance. In some industrial scenarios, this tolerance may end up being of the order of the grainsize contrast between the two species. In other cases, for example in most natural granular flows, there is a broad range of sizes over many orders of magnitudes. An interesting open question, which was not necessarily answered clearly by existing theories, is how continuous polydispersity can affect the time required for complete segregation (Fig. 1).

Segregation along inclined planes is thought to be a result of a mechanism known as kinetic sieving (Savage and Lun 1988). As the particles roll down-slope, void spaces are created by collisions. Since a small particle is more likely to fit into a void than a large particle, we observe a net movement of small particles downwards through the bulk and a corresponding net movement of large particles upwards. The speed of this movement is termed the percolation velocity, w.

In the field of kinetic sieving, two main theories are those described by Savage and Lun (1988) and Gray and Thornton (2005). They both describe thin, rapidly flowing avalanches of bidisperse mixtures down an inclined plane. Savage and Lun use an information entropy argument to develop a solution which describes the concentration profile. Gray and Thornton use a binary mixture theory to find concentration shocks which define their solutions.

Both theories include a mean segregation velocity that dictates the rate at which segregation occurs. Gray and Thornton have defined the mean segregation velocity as $q^{GT} = \pm\frac{B}{c} g\cos(\theta)$, where θ is the inclination of the slope, c is the interparticle drag and B is a dimensionless parameter. Savage and Lun proposed a different, but related equation of the form $q^{SL} = D_a\dot{\gamma}(\tilde{q}_b - \tilde{q}_a)$, where D_a is the diameter of the large particles, $\dot{\gamma}$ is the shear strain rate and $\tilde{q}_a, \tilde{q}_b$ are non-dimensional volume averaged velocities of the two constituents in the down-slope direction.

Gray and Thornton use a dimensionless parameter to indicate the specific initial conditions of the problem, the segregation number $S_r = \frac{q^{GT}L}{UH}$, where L, H and U are the typical avalanche length, thickness and down-slope velocity magnitudes respectively.

The percolation velocity of a particular species with volumetric concentration ϕ is defined by w, and for the Gray and Thornton solution is described by

$$w^{GT} = \pm \frac{B}{c} g \cos(\theta)(1 - \phi)$$

More recently, May et al. (2010) have investigated the problem by combining the segregation velocities proposed by Gray and Thornton (2005) and Savage and Lun (1988) and using

$$w^{May} = \dot{\gamma}(z) w^{GT}$$

where $\dot{\gamma}(z)$ is a function of the height only and has to be assumed within the context of May et al's theory. Here we have used the specific form of Gray, Thornton and May's proposed convex flux $f(\phi) = \phi(\phi - 1)$ (Gray and Thornton 2005; May et al. 2010).

Khakhar et al. (1999) investigate not only the role of differing particle size, but also particle density between species, again in a bi-mixture. By comparing their analytic work with numerical simulations, they make progress towards unifying a theory between two modes of segregation in natural inclined granular flows.

The above theories are restricted to bi-mixtures with moderate grainsize contrast, and neglect the effect of spontaneous percolation, whereby particles much smaller than the average can sink rapidly to the base of a flow. Most theories do not provide insight on how the size contrast between the species should affect the segregation time. An exception is given by Savage and Lun (1988), which motivates a mathematical form where the size contrast tends to enhance the segregation speed. We believe that a generalised polydisperse segregation theory can resolve this question in the most insightful way.

The solution we will present (only briefly) has the advantage of resolving $\dot{\gamma}$ as a function of height and time towards a steady state (Marks et al., Unpublished manuscript). The inclusion of shear rate in the percolation velocity is chosen to account for the rate of creation of voids which dictate the kinetic sieving mechanism. For inclined plane flow, a Bagnold velocity profile of the form $\dot{\gamma} \propto \sqrt{H - z}$ is representative of the equivalent mono-disperse system (MiDi 2004). A more realistic assumption for two species is a modified Bagnold velocity profile such as in Rognon et al. (2007).

The form of the flow equations following the method first described by Gray and Thornton (2005) are invariably some form of hyperbolic partial differential equation. These equations are notoriously difficult to solve both numerically and analytically (LeVeque 2002). We expect discontinuities (or in this case concentration shocks) to form in our solutions, but we are hampered considerably by a lack of simple solution tools to evaluate new theories.

One rather successful method of investigation has been to describe the problem as a cellular automaton (Marks and Einav 2011). This very rudimentary model can resolve a first order solution to the problem while retaining the essential physics. This method is attractive as it handles discontinuities naturally and without qualms. It has been used to create the images for this chapter. With increasing computational time, these stochastic simulations approach the smooth behaviour of the partial differential equations.

For a conservation equation to model polydisperse segregation, a number of improvements are needed to be implemented. Firstly, the idea of a continuous internal variable (Ramkrishna 2000) representing the grainsize s has to replace the hard-wired two phases in the existing models. This allows us to represent the solid fraction of a particular phase as $\phi(s)$. Secondly, each grainsize has to have its own density $\rho(s)$. Most importantly, conservation of momentum has to be used to derive a percolation velocity that explicitly includes shear rate $\dot{\gamma}$ and some function of s and $\rho(s)$ that accounts for the actual particle size and density. In Marks et al. (Unpublished manuscript) we arrive at:

$$w(z, s, t) = \dot{\gamma}(z, t) \left(\frac{s}{\bar{s}(z, t)} - \frac{\rho(s)}{\bar{\rho}(z, t)} \right) \frac{g \cos \theta}{C}$$

This form of the percolation velocity w reduces our number of fitting parameters down to one, namely C the co-efficient of interparticle friction (note that it is now dimensionless).

It was shown in Rognon et al. (2007) that there is a strong dependence of $\dot{\gamma}$ on the grain size distribution. We quantify this by including a shear rate dependent constitutive equation such as that described in MiDi (2004), concluding that an appropriate shear rate can most simply be described by

$$\dot{\gamma}(z, t) = \frac{\sqrt{3g \cos \theta} (\tan \theta - \mu_c)}{\sqrt{\bar{s^2}(z, t) 4\pi}} \sqrt{H - z}$$

Here μ_c is the critical shear parameter at which the onset of flow occurs. Results are shown for this model in a bidisperse case in Fig. 4, and a polydisperse case with a size difference of four times between the smallest and largest particle sizes in Fig. 5.

Figure 2 shows how, using w^{GT}, the time for complete segregation does not vary with the initial concentration of small particles. Figure 3 shows how using w^{May}, large particles move faster than the slower particles, as described in Marks and Einav (2011) as being a result of the depth dependent shear rate.

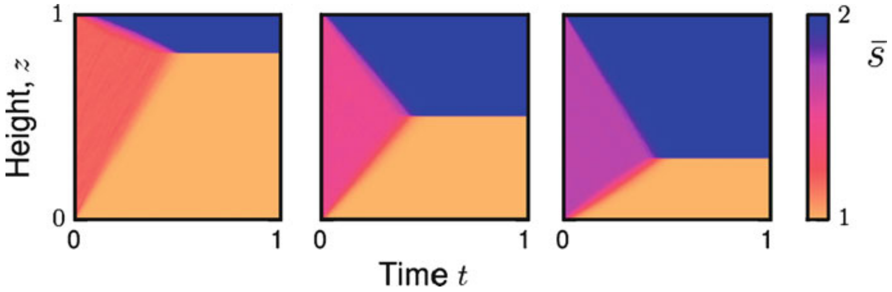

Fig. 2 Time evolution for inclined plane flow in one spatial dimension using w^{GT}. The system is initially filled with a bidisperse mixture with 30%, 50% and 80% (*left* to *right*) large concentration by volume. *Colourbar* represents the large particle concentration ϕ

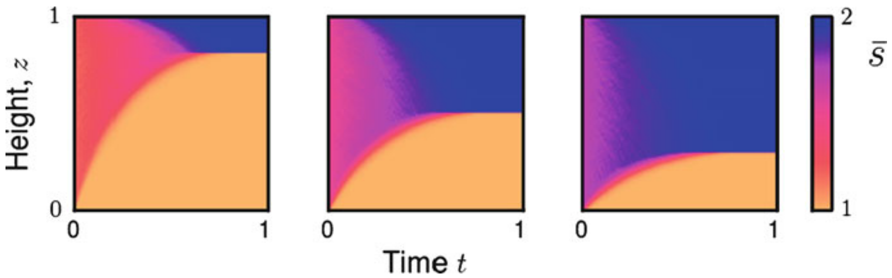

Fig. 3 Time evolution for inclined plane flow in one spatial dimension using w^{May} and $\dot{\gamma} = \sqrt{H - z}$. The system is initially filled with a bidisperse mixture with 30%, 50% and 80% (*left* to *right*) large concentration by volume. *Colourbar* represents the large particle concentration ϕ

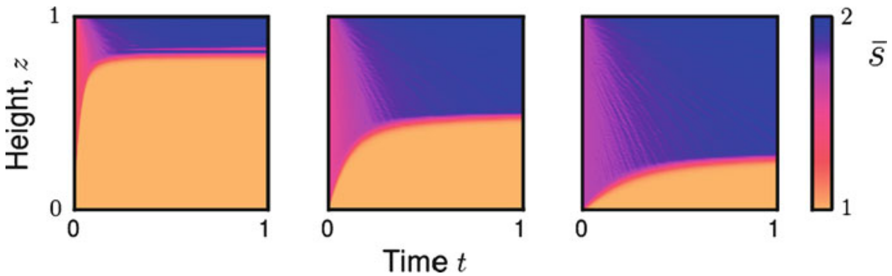

Fig. 4 Time evolution for inclined plane flow in one spatial dimension using $w(z, s, t)$ and $\dot{\gamma}(z, t)$. The system is initially filled with a bidisperse mixture with 30%, 50% and 80% (*left* to *right*) of size $s = 2$. The remainder has size $s = 1$. *Colourbar* represents the average particle size $\bar{s}$

As seen in Fig. 4, the time for complete segregation is now distinctly a function of the initial particle size distribution, even for a bi-mixture. Increasing the concentration of small particles decreases the time for segregation. For a polydisperse mixture, the time for segregation actually varies with size range. This is encapsulated in the percolation velocity, w, which now is different for each particle size.

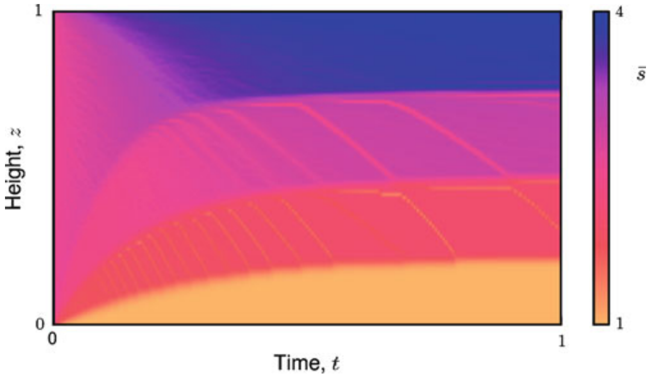

Fig. 5 Time evolution for inclined plane flow in one spatial dimension using $w(z, s, t)$ and $\dot{\gamma}(z, t)$. The system is initially filled with a polydisperse mixture of sizes varying from $s = 1$ to $s = 4$ and has volume fraction spread uniformly between these two values. *Colourbar* represents the average particle size $\bar{s}$

Each grainsize creates its own shock wave as shown in Fig. 5. These waves split the solution into regions of differing, uniform, particle size. At steady state, the particles are stacked in descending order of size so that the smallest particle is at the bottom of the flow, and the largest at the top.

The polydisperse framework captures the behaviour of particles with quantified size and density contrast, and describes the evolution of the grain size distribution at any point in space. These initial 1D numerical solutions already show the significance of the polydisperse description in terms of the grain size dependent shear strain rate, the time for complete segregation and its dependence on the initial grain size distribution.

With 2D simulations, natural avalanches and industrial mixing processes may be understood, modelled and optimised in a cohesive framework for the first time.

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