Grainsize Evolution in Open Systems

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Abstract Granular flows are often characterised by spatial and temporal variations in their grainsize distribution. These variations are generally measured by geologists and geotechnical engineers after a flow has occurred, and two limiting states are commonly found; either a power law or log-normal grainsize distribution. Here, we use a lattice model to study how the grainsize distribution evolves in granular systems subject to grain crushing, segregation and mixing simultaneously. We show how the grainsize distribution evolves towards either of these grainsize distributions depending on the mechanisms involved in the flow.

1 Introduction

There exist a large number of physical processes in which granular materials advect in space, and simultaneously undergo changes to their grainsize distribution—whether spatial rearrangement, or by changing the physical size of constituent particles. These include geophysical flows, such as avalanches, rock falls, landslides (both submarine and subaerial), mud flows, pyroclastic flows, earthquake faulting and debris flows. Such flows are also present in many industrial processes aiming at size reduction (comminution), mixing, or de-mixing of powders, grains and ores.

Spatially, grains can either mix (Utter and Behringer 2004) or de-mix (for example size segregation is a relatively well studied de-mixing phenomenon) (Savage and Lun 1988). Spatial changes in the grainsize distribution over time have received significant attention from the granular materials community recently, generally for

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K.-T. Chau and J. Zhao (eds.), *Bifurcation and Degradation of Geomaterials in the New Millennium*, Springer Series in Geomechanics and Geoengineering,
DOI 10.1007/978-3-319-13506-9_6

the particular case of bi-mixtures (Savage and Lun 1988; Dolgunin and Ukolov 1995; Gray and Thornton 2005; Gray and Chugunov 2006).

Comminution has also been a hot topic of study, but in general this study has been limited to closed systems where particles are prohibited from advecting in space (Steacy and Sammis 1991; McDowell et al. 1996). This is very rarely the case in experiment, industry, or in the field. When particles crush, the void spaces near the crushed sites rearrange. This can cause compaction in some areas, which must inevitably cause dilation in others.

In a general sense, there exists a fundamental problem with comminution modelling in that for any crushing event, we expect a change in the local arrangement of neighbours surrounding the crushing site. Conversely, local crushing events are often attributed to different modes of fracture—which are dependent on the loading state of individual particles, and strongly dependent on the local arrangement of particles. If particle arrangement is of paramount importance, then for any continuum approach, there is a fundamental issue which needs to be addressed: How does one characterise the local arrangement of particles, well below the resolution of the continuum? In this work, previous comminution models (Steacy and Sammis 1991; McDowell et al. 1996) are extended to allow open systems to be studied, in conjunction with simple models of mixing and segregation (Dolgunin and Ukolov 1995; Gray and Thornton 2005; Gray and Chugunov 2006; Marks and Einav 2011; Marks et al. 2012).

2 Continuous Open Systems

We will describe here the evolution of a system of grains with no interstitial fluid, intrinsically involving grainsize distribution at every point in space. For any given volume V in space $\underline{r}= \{x, y, z\}$ and time t, there exists a subset which is filled with grains V_s. We can then describe the volume fraction Φ that is filled by a given grainsize range $(s_a, s_b]$ as $\Phi(s_a < s \leq s_b, \underline{r}, t) = V(s_a < s \leq s_b, \underline{r}, t)/V_s(\underline{r}, t)$, where $V(s_a < s \leq s_b, \underline{r}, t)$ is the volume occupied by the given grainsize range. Furthermore, we can define a probability density function $\Phi(s, r, t)$ such that its' integral over the grainsize coordinate is the volume fraction, $\Phi(s_a < s \leq s_b, \underline{r}, t) = \int_{s_a}^{s_b} \phi(s, \underline{r}, t) ds$. With these definitions, it is possible to define conservation of mass of such a system (Ramkrishna 2000), given that it is homogeneous (constant density ρ), as

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \underline{u}) = h_b + h_d,$$

where $u(s, r, t)= \{u, v, w\}$ is the velocity of the material, $h_b(s, \underline{r}, t)$ is the birth rate, and $h_d(s, \underline{r}, t)$ is the death rate. These rates could, for instance, represent the birth of new particles of a given grainsize due to the death (fracture) of a larger particle. In this case, for some breakage rate $b\ (s, \underline{r}, t)$, the death rate could be expressed simply as

$$h_d = b\phi$$

and the corresponding birth rate would involve a summation over the deaths of all particles larger than a given size, as

$$h_b = \int_s^\infty P(s|s')b(s', \underline{r}, t)\phi(s', \underline{r}, t)ds',$$

where $P(s|s')$ is the probability that a volume of particles of grainsize s will fragment into grainsize s'. In a similar manner to conservation of mass, it is possible to define conservation of momentum for the same system (Dolgunin and Ukolov 1995; Gray and Thornton 2005; Gray and Chugunov 2006; Marks et al. 2012) such that

$$\rho \frac{D}{Dt}(\phi \underline{u}) = -\phi f \nabla \circ \sigma + \rho \phi \underline{g} - \frac{\rho \phi c}{\dot{\gamma}} \hat{\underline{u}} - d\nabla\phi,$$

where σ($\underline{r}$; t) is the size independent stress field, *f*(*s, Φ* is some grainsize dependent scaling of the stress, g is the acceleration due to gravity,) $\dot{\gamma}$ is the shear strain rate, c a coefficient of inter-particle friction, $\hat{u}(s, \underline{r}, t) = \underline{u}(s, \underline{r}, t) - \int \underline{u}\phi ds$ is the segregation velocity and d the diffusivity. This can be integrated over the full grainsize direction to describe the motion of the bulk (barycentric) flow, as $\rho \frac{D\overline{u}}{Dt} = -\nabla \cdot \sigma + \rho \underline{g}$, where we have required that $\int \phi f ds = 1$ so that conservation of bulk momentum satisfies the standard conservation law. Assuming that there is no bulk flow normal to the slope and that the segregative flow is slow, conservation of segregative momentum can be used to find the segregation velocity, $\hat{w}$, normal to the flow direction as

$$\hat{w} = |\dot{\gamma}| \frac{g \cos\theta}{c} (\frac{s}{\bar{s}} - 1) - \frac{d|\dot{\gamma}|}{c\phi} \nabla\phi,$$

where we have assumed that $f(s, \phi) = s/\bar{s}$, *and* $\bar{s} = \int \phi s ds$. We have considered here three mechanisms for grainsize change—mixing, segregation and crushing, as continuum phenomena. We will now present their analogous formulations as discrete phenomena in a simple lattice model.

3 Discrete Open Systems

3.1 Mixing

For the case of mixing of grains, there exists a strong, well established connection between the uncorrelated motion of individual constituents in a mixture, and large scale mixing of the system. This has been modelled extensively in a large variety of ways, and here we wish to describe a simple, two dimensional lattice to replicate this behaviour. Consider a system made of *NxM* cells, with position

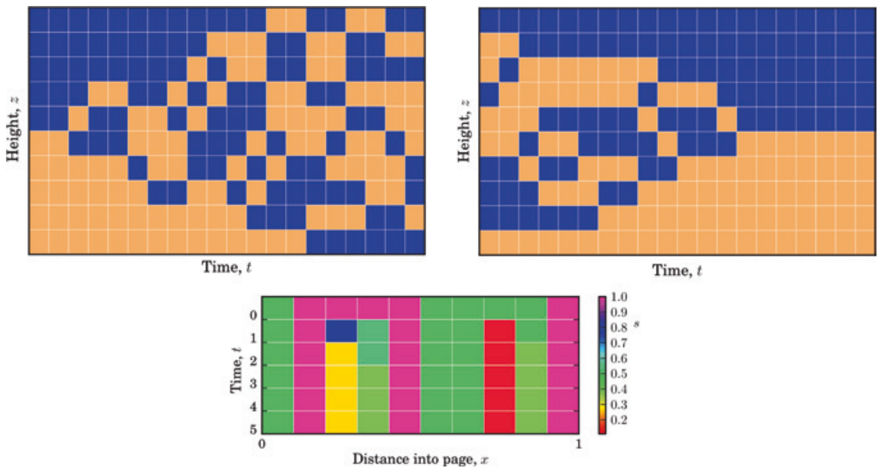

Fig. 1 *Top left* The mixing lattice model. Initially two phases are separated spatially, but over time they mix together. *Top right* The segregation lattice model. Initially two phases are well mixed (*blue* is large and *yellow* is small). Over time the two species de-mix such that the larger particles are spatially above the smaller ones. *Bottom* The crushing lattice model. Initially a bidisperse mixture is composed of two phases which crush over time such that local neighbours are of different size

$i, j = \{1 \ldots N,\ 1 \ldots M\}$ of equal volume, each filled by particles of a single grainsize $s_{i,j}$. The i direction represents a spatial coordinate, such as the perpendicular distance from the base of a flow towards the free surface. The j direction represents an internal coordinate which can be averaged over to describe the grainsize distribution. For this system, we can evolve in discrete time steps Δt. At each time, for each location $\{i, j\}$, a coin is flipped. Depending on the result, the grainsize is swapped with the cell either above or below it; Flip a coin, if heads: $s_{i,j} \Leftrightarrow s_{i-1,j}$, if tails: $s_{i,j} \Leftrightarrow s_{i+1,j}$. An example of this lattice model is shown in Fig. 1. By choosing an appropriate rate of swapping, this model is equivalent to Fickean diffusion (Chopard and Droz 1991; Utter and Behringer 2004), described by

$$\frac{\partial \phi}{\partial t} = \frac{d|\dot{\gamma}|}{c} \frac{\partial^2 \phi}{\partial z^2}.$$

3.2 Segregation

Segregation also describes advection, but now at a rate that is grainsize dependent (Savage and Lun 1988). We can capture this in a lattice model by swapping in a direction that depends on the local mean grainsize. Our rule of segregation is then:

$$\text{If}\left(s_{i,j} < \overline{s_{i,j}}\right) \wedge \left(s_{i,j} < s_{i-1,j}\right): \quad s_{i,j} \Leftrightarrow s_{i-1,j}.$$

$$\text{If}\left(s_{i,j} < \overline{s_{i,j}}\right) \wedge \left(s_{i,j} > s_{i+1,j}\right): \quad s_{i,j} \Leftrightarrow s_{i+1,j}.$$

Figure 1 shows an example of segregation, where an initially well mixed bidisperse system separates so that all of the large particles are arranged above to smaller ones. By carefully choosing the grainsize dependence of the rate of swapping, we can use this lattice model as a discrete representation for the following continuum equation,

$$\frac{\partial \phi}{\partial t} = \frac{g \cos \phi}{c} \frac{\partial}{\partial z} (\phi |\dot{\gamma}| (\frac{s}{\bar{\bar{s}}} - 1).$$

3.3 *Crushing*

Many cellular automata have been motivated to describe crushing in granular materials, but generally only for closed systems (Steacy and Sammis 1991; McDowell et al. 1996). They typically rely on two competing mechanisms to facilitate realistic behaviour. Firstly, smaller particles are more difficult to crush—because of their inherent smaller volume, they contain less large cracks, increasing their crushing stress (Weibull 1951). Secondly, particles which are surrounded by neighbours of a similar size have in general a lower coordination number, and are therefore more likely to crush due to the anisotropy of their loading condition. These two mechanisms are included in the following simple rule for crushing

$$\text{If} \left| s_{i,j} - \overline{s_{i,j}} \right| \leq \beta s_{i,j} : \quad s_{i,j}(t + \Delta t) = X s_{i,j}(t),$$

where β is a non-dimensional coefficient controlling the crushability, and X represents an i.i.d. random variable drawn uniformly from the interval (0,1]. Using this rule we can obtain an example such as that pictured in Fig. 1. Other simple numerical models have shown that the grainsize distribution evolves towards a power-law from such rules (Steacy and Sammis 1991; McDowell et al. 1996).

Each of these mechanisms alters the grainsize distribution either spatially or locally. In the future, the choice of a lattice model as the basis for the model will allow us to simply combine these rules, and observe possible complex interactions which may arise.

4 Conclusion

A continuum description for the evolution of the grainsize distribution in open systems has been developed for the three mechanisms of mixing, segregation and crushing. Analogous lattice models have also been described, and examples of their operation have been shown. These complimentary views of open systems give insight into the evolution of the complex phenomena present in many industrial and geophysical granular flows.

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